Cyclotron resonance line shape function from the equilibrium density projection operator technique

Joung Young Sug,* Sang Gyu Jo, and Sang Don Choi
Department of Physics, Kyungpook National University, Taegu 702-701, Seoul, Republic of Korea
(Received 20 April 1999; revised manuscript received 27 July 1999)

In this work, we present the calculation processes for obtaining the scattering factor for an electron-phonon system using the equilibrium density projection operator technique. We introduce two useful identities necessary to expand the scattering factor. We derive a cancellation relation which simplifies the expansion of the scattering factor. We obtain the cyclotron resonance line shape function in the case of weak interaction. Finally, we compare our results with other theories. [S1063-651X(99)13612-2]

PACS number(s): 02.50.−r, 05.40.−a

I. INTRODUCTION

Research in quantum transport theory is very important in investigating the microscopic scattering phenomena of many body systems. From 1950 to the present, many researchers have intensively carried out about this research [1–14]. Some of these researchers pointed out two problems in this field. One is the problem of nonlinear behavior under a moderately strong external field system, and the other concerns divergence in the expansion of the scattering factor at the resonance peak. Our group introduced a response function to understand these problems, utilizing the equilibrium density projection technique (EDPT) [9].

The conductivity formalism based on the Liouville equation has been successful in dealing with cyclotron transition phenomena in many works [6–8,10]. In these theories, since the conductivity function has a Lorenzian form, the scattering factor in the denominator of the conductivity function represents the inverse of the relaxation time or the line shape function of the optical absorption power. These are important to understand microscopic properties of semiconductors, etc. Thus these authors introduced many theories with various methodologies concerning the line shape formulas. Among these are Mori and Kawabata's line shape formula of an electron-impurity interacting system utilizing a projection operator based on Kubo's inner product [6], Lodder and coworkers' line shape formula of an electron-phonon interacting system utilizing a diagram method, and Suzuki's and Dunn's line shape formula of an electron-phonon interacting system utilizing a superoperator method [7]. Suzuki's result is compared with Kobori's experimental research, the result corresponding well to the experimental data in some semiconductor [11]. In this work, we summarize the calculation processes for obtaining the scattering factor using the EDPT [9], and obtain a line shape formula for an electron-phonon interacting system.

The EDPT theory represents a compact form utilizing upper nonlinear order and lower continued fractional representation (CFR) order in the derivation of the basic formula. Then we expand the form, and reorganize the calculation processes. The, CFR has a complicated interacting term, and thus we rearrange the calculation processes at first. While arranging the calculation processes, we introduce two useful identities necessary to expand the elements of the scattering factor. We also derive a cancellation relation which simplifies the expansion of the scattering factor. We explain these definitions and relation in later sections.

Most of our researches used Kubo's identity and diagonal approximation to obtain the line shape function, utilizing the combined projection technique (CPT) [10]. In the EDPT [9], we showed the difference between the scheme of the CPT and the ensemble average projection technique (EAPT). The EAPT has an advantageous aspect because it can directly obtain the line shape function, and shows the dependence on temperature of the line shape function, and the dependence on magnetic field of the line shape function and other functions; it is needed to calculate the absorption power for obtaining the linewidth in the CPT. So, in this work, we use the EAPT in the CFR formula. We strictly use the commutation relation of annihilation and creation operators and do not use the Kubo identity.

Since the perturbation of an external field is weak in the electron-phonon intraband transition between Landau levels in a semiconductor, we neglect the nonlinear effect. So, in this work, we study only the linear response term in the EDPT. Considering the weak electron-phonon interactions, we calculate the first and second terms with pair interactions in the CFR, then compare our results with other theories.

II. SYSTEM AND CONDUCTIVITY FORMULA

A. System

We consider a system of many electrons which interacts weakly with background phonons. For a static magnetic field $\vec{B}$ applied in the z direction, the electron energy is quantized. We also consider a system which is subject to an oscillatory external field $\vec{E}(t)=\hat{e}_l E_l e^{-i\omega t}$, where $\hat{e}_l$ is the unit vector in the external field direction ($l=x,y,z$, etc.), and ω is the angular frequency. Then the Hamiltonian $H(t)$ is given by

$$H(t)=H_s+H_{ext}(t)=H_s+H' E_l(t), \tag{2.1}$$

where the time-independent Hamiltonian H_s is

*FAX: 82-53-952-1739.
Electronic address: jysug@bh.knuhep.kyungpook.ac.kr

$$H_s = H_e + H_P + V, \tag{2.2}$$

$$H_e = \sum_\beta \langle \beta | h_0 | \beta \rangle a_\beta^+ a_\beta, \tag{2.3}$$

$$H_P = \sum_q \hbar \omega_q b_q^+ b_q, \tag{2.4}$$

$$V = \sum_q \sum_{\alpha,\mu} C_{\alpha,\mu}(q) a_\alpha^+ a_\mu (b_q + b_{-q}^+). \tag{2.5}$$

Here H_e is the electron Hamiltonian, H_p is the phonon Hamiltonian, V is the electron-phonon interaction Hamiltonian and a_α^+ (a_α) is the creation (annihilation) operator for electron in the Landau state $|\alpha\rangle \equiv |N_\alpha, k_{y\alpha}\rangle$, where N_α and k_α are the Landau level index and the electron wave vector, respectively. Also, m^* is the effective mass of the electron, and $\omega_c (\equiv eB/m^*)$ is the cyclotron resonance frequency of the electrons. In Eq. (2.4), b_q^+ (b_q) is the creation (annihilation) operator for a phonon in the state $|\pm q\rangle$, $\hbar\omega_q$ the phonon energy, $\vec{q}$ the phonon wave vector, and C_q the coupling coefficient for the electron-phonon interaction. The time-dependent Hamiltonian H_{ext} is

$$H_{ext}(t) = \frac{e\vec{p}}{m} \cdot \vec{A}(t) = -\vec{J} \cdot \vec{A}(t)$$
$$= -(i/\omega)\vec{E}(t) \cdot \vec{J} = H' E_l(t), \tag{2.6}$$

where

$$H' = \left(-\frac{i}{\omega}\right) J^+. \tag{2.7}$$

Here J^+ is the induced directional current component of $\vec{J}$. The many body current operators are $J^+ = \Sigma_\alpha j_\alpha^+ a_{\alpha+1}^+ a_\alpha$, $J^- = \Sigma_\alpha (j_\alpha^+)^* a_\alpha^+ a_{\alpha+1}$, and the single electron current operators are $j^\pm \equiv j_x \pm i j_y$. The corresponding Liouville operator of this system, $L(t)$, is given by

$$L(t) = L_s + L'(t). \tag{2.8}$$

Using the Landau gauge $\vec{A} = (0, Bx, 0)$, the single electron Hamiltonian is

$$h_0 = \sum_{i=x,y,z} (\vec{p} + e\vec{A})^2 / 2m_i, \tag{2.9}$$

where m_i is the effective mass in direction i. The single electron eigenstates and eigenvalues are given by

$$\epsilon_{N_\alpha, k_{z\alpha}} = \left(N_\alpha + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_{z\alpha}^2}{2m_l}, \tag{2.10}$$

$$\Psi_{N_\alpha, k_{y\alpha}, k_{z\alpha}}(x,y,z) = \frac{1}{\sqrt{L_y L_z}} \exp[i(k_{y\alpha} y + k_{z\alpha} z)] \Phi_{N\alpha}(x), \tag{2.11}$$

$$\Phi_{N\alpha}(x) = \frac{1}{\sqrt{2^{N_\alpha} N_\alpha! r_0 \sqrt{\pi}}} \times \exp[-(x - x_\alpha)^2 / 2r_0^2] H_{N\alpha}\left(\frac{x - x_\alpha}{r_0}\right), \tag{2.12}$$

where $H_N(x)$ is the Hermite polynomial, $L_y(L_z)$ is the $y(z)$ directional normalization length, N_α is the index of the Landau level, $r_0 = \sqrt{\hbar/eB}$, and $x_\alpha = \hbar k_{y\alpha}/eB$. The single current eigenvalue in this system is $j_\alpha^+ \equiv \langle \alpha+1 | j^+ | \alpha \rangle = -ie\sqrt{2(N_\alpha+1)\hbar\omega_c/m_t}$. Here m_t (m_l) is the tangent (parallel) directional effective mass about the magnetic field.

B. Conductivity formula

For the EAPT in the EDPT [9], we replace the dynamic variable operator R_k by an induced current J^- which is caused by an external circularly polarized field. The expectations of induced current and conductivity in the EAPT are

$$\tilde{J}^-(z) = \tilde{\sigma}_\mp(z) \tilde{E}_l(z), \tag{2.13}$$

$$\tilde{\sigma}_\mp(z) = \frac{-(i/\hbar)\Lambda_\mp}{iz - A_\mp + \tilde{Q}_\mp(z)}, \tag{2.14}$$

where

$$\Lambda_\mp \equiv \mathrm{Tr}\{J^- D_0\}, \tag{2.15}$$

$$A_\mp \equiv \frac{-i}{\hbar \Lambda_\mp} \mathrm{Tr}\{R_{k1} D_0\}, \tag{2.16}$$

$$Q_\mp(t) = \frac{1}{\hbar^2 \Lambda_\mp} \mathrm{Tr}\{R_{k1} G_{k0}(t) P'_{k0} L_s D_0\}. \tag{2.17}$$

Here $R_{k1} \equiv J^- L_s$, $D_0 \equiv L' \rho_s$, $G_{k0}(t) = \exp(-itP'_{k0} L_s/\hbar)$, P_{k0} is Eq. (3.2), and $P'_{k0} = 1 - P_{k0}$.

Using the matrix elements of Sec. III, we calculate $\Lambda_\mp$ and $A_\mp$ as follows:

$$\Lambda_\mp = \left(\frac{i}{\omega}\right) \sum_\alpha j_\alpha^{+*} j_\alpha^+ (f_{\alpha+1} - f_\alpha), \tag{2.18}$$

$$A_\mp = -i\omega_c. \tag{2.19}$$

In the continuous approximation for the electron state, we need the following replacements:

$$\sum_\alpha \Rightarrow \frac{m_t \omega_c}{2\pi^2 \hbar} \sum_{N=0}^{\infty} \int_{-\infty}^{\infty} dk_{z\alpha}, \tag{2.20}$$

$$\sum_\beta \mathrm{Tr}^{(e)}\{\rho(H_e) a_\alpha^+ a_\beta\} = \frac{m_t \omega_c}{2\pi^2 \hbar} \sum_{N=0}^{\infty} \int_{-\infty}^{\infty} f_\alpha dk_{z\alpha} \delta_{\alpha,\beta}. \tag{2.21}$$

With these replacements, we obtain

$$\Lambda_{\mp}=\frac{m_t\omega_c}{2\pi^2\hbar}\left(\frac{i}{\omega}\right)j_\alpha^{+2}\sum_{N=0}^{\infty}\int_{-\infty}^{\infty}dk_{z\alpha}(f_{\alpha+1}-f_\alpha), \tag{2.22}$$

where z is replaced by the external oscillatory frequency $-\omega$. Then we obtain a formula for the conductivity tensor in this theory:

$$\tilde{\sigma}_{\mp}(\omega)=-\frac{m_t\omega_c}{2\pi^2\hbar}\sum_{N=0}^{\infty}\frac{i}{\hbar\omega}\frac{j_\alpha^{+2}\int_{-\infty}^{\infty}dk_{z\alpha}(f_\alpha-f_{\alpha+1})}{\omega-\omega_c+i\tilde{Q}_{\mp}(\omega)}. \tag{2.23}$$

Our formula for the conductivity tensor in Eq. (2.23) is similar to those of many other theories. However, our scattering factor $\tilde{Q}_{\mp}(\omega)$ is quite different from others [5–8,10].

III. REGULATIONS IN EXPANDING THE ELEMENTS OF CFR FACTORS

We expanded the scattering factor $\tilde{Q}_{\mp}(\omega)$ in a continued fraction representation to avoid divergence at the resonance peak, and expanded the CFR formula again in a series form in order to examine the convergence [9,12]. Taking a diagonal approximation and assuming weak electron-phonon interactions, we can approximately separate the background phonon average for the scattering factor of linear response function, $\tilde{Q}_{\mp}(\omega)$, as

$$\tilde{Q}_{\mp}(\omega)\approx\frac{1}{\hbar^2\Lambda_{\mp}}\langle\mathrm{Tr}^e\{R_{k1}f_1(\omega)\}\rangle_B. \tag{3.1}$$

Here, Tr^e denotes the trace of electron states and $\langle\cdots\rangle_B$ denotes the ensemble average of the background. $f_1(\omega)$ is the Fourier-Laplace transform of $f_1(\tau_1)\equiv e^{-i\tau_1L_1/\hbar}f_1$, where $\tau_1\equiv t-s$. We define the projection operator P_0 by

$$P_0X\equiv\frac{D_e}{\Lambda_{\mp}}\mathrm{Tr}^{(e)}\{J^-X\}, \tag{3.2}$$

with $D_e\equiv L_+\rho_e$, $f_1\equiv L_1D_e$, and $L_1\equiv(1-P_0)L_s\equiv P_0'L_s$.

If we assume that the electron-phonon interactions are weak enough, it will suffice to consider only the two lowest order terms in the series expansion of the CFR as

$$\tilde{Q}_{\mp}(\omega)\approx i\gamma_0+(i\Delta_1)K_1. \tag{3.3}$$

Here we expand the elements of scattering factor as in Ref. [9], as follows:

$$\gamma_0=\frac{-1}{(-\omega)\hbar^2\langle\mathrm{Tr}^{(e)}\{J^-L_l'\rho_e\}\rangle_B}\langle\mathrm{Tr}^{(e)}\{J^-L_sP_0'L_sL_+'\rho_e\}\rangle_B, \tag{3.4}$$

$$\Delta_1\equiv\frac{i}{(-\omega)\hbar^3\langle\mathrm{Tr}^{(e)}\{J^-L_+'\rho_e\}\rangle_B}\times\langle\mathrm{Tr}^{(e)}\{J^-L_sP_0'L_sP_0'L_sL_+'\rho_e\}\rangle_B, \tag{3.5}$$

$$K_1=\frac{1}{i(-\omega)+i\omega_1+i\gamma_1}, \tag{3.6}$$

$$\omega_1=\frac{-1}{\hbar\langle\mathrm{Tr}^{(e)}\{J^-L_sP_0'L_sP_0'L_sL_+'\rho_e\}\rangle_B}\times\langle\mathrm{Tr}^{(e)}\{J^-L_sP_0'L_sP_0'L_sP_0'L_sL_+'\rho_e\}\rangle_B, \tag{3.7}$$

$$\gamma_1\equiv\alpha+\frac{\omega_1^2}{(-\omega)}, \tag{3.8}$$

$$\alpha\equiv\frac{-1}{(-\omega)\hbar^2\langle\mathrm{Tr}^{(e)}\{J^-L_sP_0'L_sP_0'L_sL_+'\rho_e\}\rangle_B}\times\langle\mathrm{Tr}^{(e)}\{J^-L_sP_0'L_s'P_0'L_sP_0'L_sP_0'L_sL_+'\rho_e\}\rangle_B, \tag{3.9}$$

$$\Lambda_{\mp}\equiv\langle\mathrm{Tr}^{(e)}\{J^-L_+'\rho_e\}\rangle_B. \tag{3.10}$$

We rearrange the operators in such a way that all Liouville operators act on J^-. We introduce a useful identity necessary to inverse the order of Liouville operators (see the Appendix).

$$\mathrm{Tr}^{(e)}\{J^-L_1L_2\cdots L_{m-1}L_mL_+'\rho_e\}=(-1)^{(m+1)}\langle L_+'L_mL_{m-1}\cdots L_2L_1J^-\rangle_e, \tag{3.11}$$

where $\langle\cdots\rangle_e$ means the ensemble average for H_e. To calculate elements of the scattering factors, we derive some useful identities necessary to adapt $P_0'=1-P_0$:

$$G0=(L2)-(B_0)(L1)(L1), \tag{3.12}$$

$$G1=(L3)-(B_0)(L1)(L2)-(B_0)(G0)(L1), \tag{3.13}$$

$$G2=(L4)-(B_0)(L3)(L1)-(B_0)(G0)(L2)-(B_0)(G1)(L1), \tag{3.14}$$

$$G3=(L5)-(B_0)(L1)(L4)-(B_0)(G0)(L3)-(B_0)(G1)(L2)-(B_0)(G2)(L1), \tag{3.15}$$

with the definitions

$$GN\equiv\mathrm{Tr}^{(e)}\{J^-L_s(P_0'L_s)^{N+1}L_+\rho_s\},\quad (N\text{ positive number}), \tag{3.16}$$

$$(Ln)\equiv\pm\langle L_+(L_s)^nJ^-\rangle_s,\quad (n\text{ odd},+;\ n\text{ even},-), \tag{3.17}$$

$$(B_0)\equiv\frac{-1}{\langle L_+J^-\rangle_s}. \tag{3.18}$$

Utilizing above identities, we rewrite the elements as follows:

$$\gamma_0=\frac{-1}{(-\omega)\hbar^2}(B_0)(G0), \tag{3.19}$$

$$\Delta_1 = \frac{i}{(-\omega)\hbar^3}(B_0)(G1), \tag{3.20}$$

$$\omega_1 = \frac{-1}{\hbar G1} G2, \tag{3.21}$$

$$\gamma_1 = \alpha + \frac{\omega_1^2}{(-\omega)}, \tag{3.22}$$

$$\alpha = \frac{-1}{(-\omega)\hbar^2 G1} G3. \tag{3.23}$$

IV. LINE SHAPE FUNCTION

In order to calculate matrix elements, we use the commutation relations of fermions (electrons) and anticommutation relations of bosons (phonons). Using the relation

$$\begin{aligned}\langle (b_l + b_{-l}^+)(b_q + b_{-q}^+)\rangle_p &= \langle (b_q + b_{-q}^+)(b_l + b_{-l}^+)\rangle_p \\ &= \{n_q + (n_q+1)\}\delta_{l,-q}, \end{aligned}\tag{4.1}$$

and assuming the weak interactions, we may take

$$\begin{aligned}&\langle [a_\nu^+ a_\kappa (b_l + b_{-l}^+), a_\mu^+ a_{\alpha+1}(b_q + b_{-q}^+)]\rangle_p \\ &\quad = [a_\nu^+ a_\kappa, a_\mu^+ a_{\alpha+1}]\langle (b_l + b_{-l}^+)(b_q + b_{-q}^+)\rangle_p. \end{aligned}\tag{4.2}$$

Substituting L_s, given by

$$L_s \equiv L_e + L_p + L_v \equiv L_d + L_v, \tag{4.3}$$

into the elements of scattering factor, we obtain the matrix elements, which are needed to obtain the line shape function, as follows:

$$\langle L'(L_p)^n J^-\rangle_s = 0, \tag{4.4}$$

$$\begin{aligned}&\langle L'(L_d)^n J^-\rangle_s \\ &\quad = \left(-\frac{i}{\omega}\right)\sum_\alpha (\epsilon_\alpha - \epsilon_{\alpha+1})^n j_\alpha^+ j_\alpha^{+*}\langle (f_{\alpha+1} - f_\alpha)\rangle_p = 0, \end{aligned}\tag{4.5}$$

$$\begin{aligned}&\langle L'(L_e)^n L_v J^-\rangle_s \\ &\quad = \left(-\frac{i}{\omega}\right)\sum_q (C_{\alpha,\alpha}(q) - C_{\alpha+1,\alpha+1}(q)) \\ &\qquad \times (\epsilon_\alpha - \epsilon_{\alpha+1})^n j_\alpha^+ j_\alpha^{+*}\langle (f_{\alpha+1} - f_\alpha)(b_q + b_{-q}^+)\rangle_s, \end{aligned}\tag{4.6}$$

$$\begin{aligned}&\langle L'(L_p)^n L_v J^-\rangle_s \\ &\quad = \left(-\frac{i}{\omega}\right)\sum_q (\hbar\omega_q)^n (C_{\alpha,\alpha}(q) - C_{\alpha+1,\alpha+1}(q)) \\ &\qquad \times j_\alpha^+ j_\alpha^{+*}\langle (f_{\alpha+1} - f_\alpha)((-1)^n b_q + b_{-q}^+)\rangle_s, \end{aligned}\tag{4.7}$$

$$\begin{aligned}\langle L'(L_d)^n L_v J^-\rangle_s = &\left(-\frac{i}{\omega}\right)\sum_q (\epsilon_\alpha - \epsilon_{\alpha+1} - \hbar\omega_q)^n \\ &\times (C_{\alpha,\alpha}(q) - C_{\alpha+1,\alpha+1}(q)) \\ &\times j_\alpha^+ j_\alpha^{+*}\langle (f_{\alpha+1} - f_\alpha) b_q\rangle_s \\ &+ \left(-\frac{i}{\omega}\right)\sum_q (\epsilon_\alpha - \epsilon_{\alpha+1} + \hbar\omega_q)^n \\ &\times (C_{\alpha,\alpha}(q) - C_{\alpha+1,\alpha+1}(q)) \\ &\times j_\alpha^+ j_\alpha^{+*}\langle (f_{\alpha+1} - f_\alpha) b_{-q}^+\rangle_s, \end{aligned}\tag{4.8}$$

$$\begin{aligned}&\langle L' L_v (L_d)^n L_v J^-\rangle_s \\ &\quad = \langle A_n + B_n + C_n + D_n + F_n + G_n + H_n + I_n\rangle_s, \end{aligned}\tag{4.9}$$

where

$$\begin{aligned}A_n \equiv &\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\beta - \epsilon_{\alpha+1} - \hbar\omega_q)^n \\ &\times C_{\alpha,\beta}(q) C_{\beta,\alpha}^+(q) j_\alpha^+ j_\alpha^{+*}(f_{\alpha+1} - f_\alpha)(n_q), \end{aligned}\tag{4.10}$$

$$\begin{aligned}B_n \equiv -&\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\beta - \epsilon_{\alpha+1} - \hbar\omega_q)^n \\ &\times C_{\alpha+1,\beta+1}(q) C_{\beta,\alpha}^+(q) j_\beta^+ j_\alpha^{+*}(f_{\beta+1} - f_\beta)(n_q), \end{aligned}\tag{4.11}$$

$$\begin{aligned}C_n \equiv -&\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\alpha - \epsilon_{\beta+1} - \hbar\omega_q)^n \\ &\times C_{\alpha+1,\beta+1}(q) C_{\beta,\alpha}^+(q) j_\beta^+ j_\alpha^{+*}(f_{\beta+1} - f_\beta)(n_q), \end{aligned}\tag{4.12}$$

$$\begin{aligned}D_n \equiv &\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\alpha - \epsilon_\beta - \hbar\omega_q)^n \\ &\times C_{\alpha+1,\beta}(q) C_{\beta,\alpha+1}^+(q) j_\alpha^+ j_\alpha^{+*}(f_{\alpha+1} - f_\alpha)(n_q), \end{aligned}\tag{4.13}$$

$$\begin{aligned}F_n \equiv &\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\beta - \epsilon_{\alpha+1} + \hbar\omega_q)^n \\ &\times C_{\alpha,\beta}^+(q) C_{\beta,\alpha}(q) j_\alpha^+ j_\alpha^{+*}(f_{\alpha+1} - f_\alpha)(n_q+1), \end{aligned}\tag{4.14}$$

$$\begin{aligned}G_n \equiv -&\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\beta - \epsilon_{\alpha+1} + \hbar\omega_q)^n \\ &\times C_{\alpha+1,\beta+1}^+(q) C_{\beta,\alpha}(q) j_\beta^+ j_\alpha^{+*}(f_{\beta+1} - f_\beta)(n_q+1), \end{aligned}\tag{4.15}$$

$$H_n \equiv -\left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\alpha - \epsilon_{\beta+1} + \hbar\omega_q)^n \times C^+_{\alpha+1,\beta+1}(q) C_{\beta,\alpha}(q) j^+_\beta j^{+*}_\alpha (f_{\beta+1} - f_\beta)(n_q+1), \tag{4.16}$$

$$I_n \equiv \left(-\frac{i}{\omega}\right)\sum_q \sum_\beta (\epsilon_\alpha - \epsilon_\beta + \hbar\omega_q)^n \times C^+_{\alpha+1,\beta}(q) C_{\beta,\alpha+1}(q) j^+_\alpha j^{+*}_\alpha (f_{\alpha+1} - f_\alpha)(n_q+1). \tag{4.17}$$

Here $\langle\cdots\rangle_s$ ($\langle\cdots\rangle_p$, etc.) means the ensemble average for H_s (H_p, etc.) and f_α is the Fermi-Diac distribution of α state. We expand GN's with the order of Liouville operators kept the same in each term as follows:

$$G0 = \langle -\langle L_+ L_s L_s J^-\rangle_e - (B_0)\langle L_+ L_s J^-\rangle_e \langle L_+ L_s J^-\rangle_e \rangle_p = G0a + G0b, \tag{4.18}$$

$$G0a \equiv \langle -\langle L_+ L_d L_d J^-\rangle_e - (B_0)\langle L_+ L_d J^-\rangle_e \langle L_+ L_d J^-\rangle_e \rangle_p, \tag{4.19}$$

$$G0b \equiv \langle -\langle L_+ L_v L_v J^-\rangle_e - (B_0)\langle L_+ L_v J^-\rangle_e \langle L_+ L_v J^-\rangle_e \rangle_p. \tag{4.20}$$

Using the matrix elements [Eqs. (4.4)–(4.9)], we obtain a useful cancellation relation which simplifies the expansion of scattering factors as

$$M_m + I_m = 0, \tag{4.21}$$

where

$$M_m \equiv \langle (B_0)\langle L_+ L_v (L_d)^m J^-\rangle_e \langle L_+ L_v J^-\rangle_e \rangle_B, \tag{4.22}$$

$$N_m \equiv \langle (B_0)(B_0)\langle L_+ L_v J^-\rangle_e \langle L_+ (L_d)^m J^-\rangle_e \langle L_+ L_v J^-\rangle_e \rangle_B. \tag{4.23}$$

Here M_m and N_m are opposite signs of each other according to Eqs. (3.17) and (3.18). Then, we obtain GN's as follows:

$$G1 = \langle \langle L_+ L_v L_d L_v J^-\rangle_e + (B_0)\langle L_+ L_v J^-\rangle_e \langle L_+ L_d L_v J^-\rangle_e \rangle_p, \tag{4.24}$$

$$G2 = G2(2\mathrm{nd}) + G2(4\mathrm{th}) \approx G2(2\mathrm{nd}),$$

$$G2(2\mathrm{nd}) \equiv \langle -\langle L_+ L_v L_d L_d L_v J^-\rangle_e - (B_0)\langle L_+ L_v J^-\rangle_s \langle L_+ L_d L_d L_v J^-\rangle_e \rangle_p, \tag{4.25}$$

$$G3 = G3(2\mathrm{nd}) + G3(\text{higher order}) \approx G3(2\mathrm{nd}),$$

$$G3(2nd) \equiv \langle \langle L_+ L_v L_d L_d L_d L_v J^-\rangle_e + (B_0)\langle L_+ L_v J^-\rangle_e \langle L_+ L_d L_d L_d L_v J^-\rangle_e \rangle_p, \tag{4.26}$$

$$(B_0) \equiv -\frac{1}{\langle L_+ J^-\rangle_s}. \tag{4.27}$$

This theory contains the terms of high order interactions in the lowest second order approximation, while most of the other theories do not contain such high order terms in the same approximation [6–8,10]. Thus, we can apply this formula to a moderately strongly interacting system in the lowest second order approximation. But the calculation of high order interactions is very complicated, and we need to develop some techniques. Here, considering the weakness of interactions, we neglect the higher order interactions (fourth orders, etc). Then we obtain

$$G0 = [A_0 + B_0 + C_0 + F_0 + G_0 + H_0]_{(\beta\neq\alpha)} + [D_0 + I_0]_{(\beta\neq\alpha+1)}, \tag{4.28}$$

$$G1 = [A_1 + B_1 + C_1 + F_1 + G_1 + H_1]_{(\beta\neq\alpha)} + [D_1 + I_1]_{(\beta\neq\alpha+1)}, \tag{4.29}$$

$$G2 \approx G2(2\mathrm{nd}) = [A_2 + B_2 + C_2 + F_2 + G_2 + H_2]_{(\beta\neq\alpha)} + [D_2 + I_2]_{(\beta\neq\alpha+1)}, \tag{4.30}$$

$$G3 \approx G3(2\mathrm{nd}) = [A_3 + B_3 + C_3 + F_3 + G_3 + H_3]_{(\beta\neq\alpha)} + [D_3 + I_3]_{(\beta\neq\alpha+1)}. \tag{4.31}$$

Thus, we obtain the line shape function $\tilde{Q}_\mp(\omega)$ as follows:

$$\tilde{Q}_\mp(\omega) = \frac{1}{\hbar^2 \Lambda_\mp}\{R_{k1} f_1(z)\} \approx i\gamma_0 + (i\Delta_1)K_1, \tag{4.32}$$

$$K_1 = \frac{1}{i\omega + i\omega_1 + i\gamma_1}, \tag{4.33}$$

$$\gamma_0 = \frac{1}{\omega\hbar^2}\{[A_0 + B_0 + C_0 + F_0 + G_0 + H_0]_{(\beta\neq\alpha)} + [D_0 + I_0]_{(\beta\neq\alpha+1)}\}, \tag{4.34}$$

$$\Delta_1 = \frac{-i}{\omega\hbar^3}(B_0)\{[A_1+B_1+C_1+F_1+G_1+H_1]_{(\beta\neq\alpha)}+[D_1+I_1]_{(\beta\neq\alpha+1)}\}, \tag{4.35}$$

$$\omega_1 = \frac{-1}{\hbar}\frac{\{[A_2+B_2+C_2+F_2+G_2+H_2]_{(\beta\neq\alpha)}+[D_2+I_2]_{(\beta\neq\alpha+1)}\}}{\{[A_1+B_1+C_1+F_1+G_1+H_1]_{(\beta\neq\alpha)}+[D_1+I_1]_{(\beta\neq\alpha+1)}\}}, \tag{4.36}$$

$$\gamma_1 = \frac{1}{\omega\hbar^2}\frac{\{[A_3+B_3+C_3+F_3+G_3+H_3]_{(\beta\neq\alpha)}+[D_3+I_3]_{(\beta\neq\alpha+1)}\}}{\{[A_1+B_1+C_1+F_1+G_1+H_1]_{(\beta\neq\alpha)}+[D_1+I_1]_{(\beta\neq\alpha+1)}\}} + \frac{1}{\omega\hbar}\left[\frac{\{[A_2+B_2+C_2+F_2+G_2+H_2]_{(\beta\neq\alpha)}+[D_2+I_2]_{(\beta\neq\alpha+1)}\}}{\{[A_1+B_1+C_1+F_1+G_1+H_1]_{(\beta\neq\alpha)}+[D_1+I_1]_{(\beta\neq\alpha+1)}\}}\right]^2. \tag{4.37}$$

The result of the line shape function [Eq. (4.32)], with Eqs. (4.33)–(4.37), has more interaction energy terms than in other theories. Some of those intraband transition theories are successful in explaining the transitions between Landau levels in some materials (Si, Ge, etc.) [11]. However, in some materials (GaAs, etc.), the theories do not explain experimental data sufficiently well. We expect that the result in Eq. (4.32) will supplement the explanation of those experimental data. In Sec. V, we compare our result with other theories.

V. LINESHIFT AND HALFWIDTH

If we let $\overline{\omega}\equiv-\omega-i\eta$, K_1 becomes complex, and we take

$$\tilde{Q}_{\mp}(\overline{\omega})\equiv iS(\overline{\omega})+\gamma(\overline{\omega}), \tag{5.1}$$

where the line shift is

$$S(\overline{\omega})\equiv I_m Q_{\mp}(\overline{\omega})\approx I_m\{i\gamma_0+(i\Delta_1)K_1\}=\gamma_0+I_m\{(i\Delta_1)K_1\}, \tag{5.2}$$

and the halfwidth is

$$\gamma(\overline{\omega})\equiv R_e Q_{\mp}(\overline{\omega})\approx R_e\{i\gamma_0+(i\Delta_1)K_1\}=R_e\{(i\Delta_1)K_1\}. \tag{5.3}$$

Substituting $|j_\alpha^{+2}|=e^2[2(N_\alpha+1)\hbar\omega_c/m_t]$ into the conductivity formula, we obtain the conductivity as

$$\mathrm{Re}\,\sigma_{\mp}(\overline{\omega}) = -\frac{e^2}{\pi^2\hbar}\left(\frac{\omega_c^2}{\omega}\right)\frac{\gamma(\overline{\omega})\int_{-\infty}^{\infty}dk_z(f_{N,k_z}-f_{N+1,k_z})}{(\overline{\omega}-\omega_c-S(\omega))^2+[\gamma(\overline{\omega})]^2}. \tag{5.4}$$

For the circularly polarized electromagnetic field with the frequency ω applied to the system, the absorption power delivered to the system is given by

$$P(\omega)=(E_0^2/2)\mathrm{Re}\{\sigma_{\mp}(\overline{\omega})\},$$

where Re denotes the real part. Since the conductivity function equation (5.4) is of Lorenzian form, the scattering factors $S(\overline{\omega})$ and $\gamma(\overline{\omega})$ in the denominator of conductivity function play the role of the line shape function of the optical absorption power. The line shape function is important to understand microscopic properties of semiconductors [6–11].

Most of the other theories require a calculation of the absorption power in order to obtain the linewidth, because the whole conductivity formula must be integrated over the electron wave vector k_z [7–10]. However, in the EAPT, the integration over the state $|\alpha\rangle$ appears separately in the numerator and denominator of the conductivity formula [9]. This scheme has advantageous aspects in that we can directly obtain the line shape function and well explain the dependence of the temperature, the dependence of the magnetic field, and so on. The interaction coupling factor C_q is

$$C_{\alpha,\mu}(q)=V_q\langle\alpha|e^{i\vec{q}\cdot\vec{r}}|\mu\rangle,$$

where V_q is the material factor in each case. For example, it can be an acoustic deformation potential scattering factor, an acoustic piezoelectric scattering factor, a polar optical phonon scattering factor, etc. The dependence of V_q on the phonon wave vector q differs in each scattering mechanism. The matrix elements $C_{\alpha,\mu}(q)$ are given by

$$(C_q)_{\alpha,\beta}(C_q^+)_{\beta,\alpha}=V(q)^2K_1(N_\alpha,N_\beta;t)\,\delta_{k_{\beta_z}k_{\alpha_z}-q_z},$$
$$(C_q)_{\alpha+1,\beta+1}(C_q^+)_{\beta,\alpha}=V(q)^2K_2(N_\alpha,N_\beta;t)\,\delta_{k_{\beta_z}k_{\alpha_z}-q_z} \tag{5.5}$$

with K matrices

$$K_1(N_>,N_<;t)\equiv\frac{N_<!}{N_>!}t^{(N_>-N_<)}\exp(-t)(L_{N_<}^{(N_>-N_<)})^2,$$
$$K_2(N_<,N_>;t)\equiv\sqrt{\frac{N_<+1}{N_>+1}}\frac{N_<!}{N_>!}t^{(N_>-N_<)}\times\exp(-t)L_{N_<}^{(N_>-N_<)}(t)L_{N_<+1}^{(N_>-N_<)}(t), \tag{5.6}$$

where

$$L_n^m(t)=(n!)^{-1}\exp(t)t^{-m}(d^n/dt^n)[t^{n+m}\exp(-t)] \tag{5.7}$$

is the associated Larguerre polynomial. Here $N_{>}$ ($N_{<}$) denotes the larger (smaller) number among N_α and N_β.

In continuous approximation for the phonon wave vector $q=\sqrt{q_x^2+q_y^2+q_y^2}$, we take the integration over phonon states as

$$\sum_q \to \Omega(2\pi)^{-3}\int_{-\infty}^{\infty} dq_z \int_0^{\infty}(2\pi/r_0^2)dt, \tag{5.8}$$

where Ω is the volume of materials, and

$$t\equiv[r_0^4 q_y^2+r_0^4 q_x^2]/2r_0^2=\frac{r_0^2}{2[q_y^2+q_x^2]}. \tag{5.9}$$

Then we obtain the integration forms of matrix elements as follows:

$$(B_0)'\equiv\frac{-1}{\int_{-\infty}^{\infty} dk_{z\alpha}(f_{\alpha+1}-f_\alpha)}, \tag{5.10}$$

$$A'_{n,(\beta\neq\alpha)}(\alpha,\beta) =[(N_\alpha+1)]\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt\, V(q)^2 \times\Delta E_n(\beta,\alpha+1;-q_z)K_1(N_\alpha,N_\beta;t)\Delta F(\alpha)n_q, \tag{5.11}$$

$$B'_{n,(\beta\neq\alpha)}(\alpha,\beta)=-[(N_\beta+1)(N_\alpha+1)]^{1/2} \times\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt \times V(q)^2\Delta E_n(\beta,\alpha+1;-q_z) \times K_2(N_\alpha,N_\beta;t)\Delta F(\beta,-q_z)n_q, \tag{5.12}$$

$$C'_{n,(\beta\neq\alpha)}(\alpha,\beta) =-[(N_\beta+1)(N_\alpha+1)]^{1/2} \times\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt\, V(q)^2\Delta E_n(\alpha,\beta+1;-q_z) \times K_2(N_\alpha,N_\beta;t)\Delta F(\beta,-q_z)n_q, \tag{5.13}$$

$$D'_{n,(\beta\neq\alpha+1)}(\alpha,\beta) =[(N_\alpha+1)]\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt\, V(q)^2 \times\Delta E_n(\alpha,\beta;-q_z)K_1(N_{\alpha+1},N_\beta;t)\Delta F(\alpha)n_q, \tag{5.14}$$

$$F'_{n,(\beta\neq\alpha)}(\alpha,\beta) =[(N_\alpha+1)]\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt\, V(q)^2 \times\Delta E_n(\beta,\alpha+1;q_z)K_1(N_\alpha,N_\beta;t)\Delta F(\alpha)(n_q+1), \tag{5.15}$$

$$G'_{n,(\beta\neq\alpha)}(\alpha,\beta) =-[(N_\beta+1)(N_\alpha+1)]^{1/2}\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt \times V(q)^2\Delta E_n(\beta,\alpha+1;q_z) \times K_2(N_\alpha,N_\beta;t)\Delta F(\beta,+q_z)(n_q+1), \tag{5.16}$$

$$H'_{n,(\beta\neq\alpha)}(\alpha,\beta) =-[(N_\beta+1)(N_\alpha+1)]^{1/2}\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt \times V(q)^2\Delta E_n(\alpha,\beta+1;q_z)K_2(N_\alpha,N_\beta;t) \times\Delta F(\beta,+q_z)(n_q+1), \tag{5.17}$$

$$I'_{n,(\beta\neq\alpha+1)}(\alpha,\beta) =[(N_\alpha+1)]\int_{-\infty}^{\infty} dk_{z\alpha}\int_{-\infty}^{\infty} dq_z\int_0^{\infty} dt \times V(q)^2\Delta E_n(\alpha,\beta;q_z)K_1(N_{\alpha+1},N_\beta;t) \times\Delta F(\alpha)(n_q+1), \tag{5.18}$$

where

$$\Delta E_n(\beta,\alpha+1;\pm q_z) \equiv\left[(N_\beta-N_\alpha-1)\hbar\omega_c+\frac{\hbar^2}{2m^*}(\pm 2k_{z_\alpha}q_z+q_z^2) \pm\hbar v_s\left(\frac{2}{r_0^2}t+q_z^2\right)^{1/2}\right]^n, \tag{5.19}$$

$$\Delta E_n(\alpha,\beta+1;\pm q_z) \equiv\left[(N_\alpha-N_\beta-1)\hbar\omega_c-\frac{\hbar^2}{2m^*}(\pm 2k_{z_\alpha}q_z+q_z^2) \pm\hbar v_s\left(\frac{2}{r_0^2}t+q_z^2\right)^{1/2}\right]^n, \tag{5.20}$$

$$\Delta E_n(\alpha,\beta;\pm q_z)\equiv\left[(N_\alpha-N_\beta)\hbar\omega_c-\frac{\hbar^2}{2m^*}(\pm 2k_{z_\alpha}q_z+q_z^2) \pm\hbar v_s\left(\frac{2}{r_0^2}t+q_z^2\right)^{1/2}\right]^n, \tag{5.21}$$

and with $\Delta F(\alpha)\equiv(f_{\alpha+1}-f_{\alpha})$, $\Delta F(\beta,\pm q_z)\equiv(f_{\beta+1}-f_{\beta})$, $f_{\alpha}\equiv[e^{\epsilon(\alpha,T)}+1]^{-1}$, $f_{\beta}\equiv[e^{\epsilon(\beta,\pm q_z,T)}+1]^{-1}$, $n_q\equiv[e^{\epsilon(q,T)}-1]^{-1}$. Here

$$\epsilon(\alpha,T)=\left[\left(N_{\alpha}+\frac{1}{2}\right)\hbar\omega_c+\frac{\hbar^2k_{z_{\alpha}}^2}{2m^*}+(\epsilon_c-\epsilon_F)\right]\Big/ k_BT, \tag{5.22}$$

$$\epsilon(\beta,\pm q_z,T)=\left[(N_{\beta}+1)\hbar\omega_c+\frac{\hbar^2(k_{z_{\alpha}}\pm q_z)^2}{2m^*}+(\epsilon_c-\epsilon_F)\right]\Big/ k_BT, \tag{5.23}$$

$$\epsilon(q,T)=\frac{\hbar\omega_q}{K_BT}\approx\frac{\hbar v_s}{K_BT}\left(\frac{2}{r_0^2t}+q_z^2\right)^{1/2}. \tag{5.24}$$

Then elements of the line shape formula with Eqs. (5.10)–(5.18) are

$$\gamma_0=\frac{-1}{\hbar^2\overline{\omega}}\left(\frac{1}{4\pi^2r_0^2}\right)(B'_{k0})(A'_0+B'_0+C'_0+D'_0+F'_0+G'_0+H'_0+I'_0), \tag{5.25}$$

$$\Delta_1=\left(\frac{i}{\hbar^3\overline{\omega}\Lambda_{kl0}}\right)\Lambda_{kl1}=\frac{i}{\hbar^3\overline{\omega}}\left(\frac{1}{4\pi^2r_0^2}\right)(B'_{k0})\times(A'_1+B'_1+C'_1+D'_1+F'_1+G'_1+H'_1+I'_1), \tag{5.26}$$

$$\omega_1=\frac{-1}{\hbar}\frac{(A'_2+B'_2+C'_2+D'_2+F'_2+G'_2+H'_2+I'_2)}{(A'_1+B'_1+C'_1+D'_1+F'_1+G'_1+H'_1+I'_1)}, \tag{5.27}$$

$$\gamma_1=\frac{-1}{\hbar^2\overline{\omega}}\frac{(A'_3+B'_3+C'_3+D'_3+F'_3+G'_3+H'_3+I'_3)}{(A'_1+B'_1+C'_1+D'_3+F'_1+G'_1+H'_1+I'_1)}+\frac{1}{\overline{\omega}}\left[\frac{-1}{\hbar}\frac{(A'_2+B'_2+C'_2+D'_2+F'_2+G'_2+H'_2+I'_2)}{(A'_1+B'_1+C'_1+D'_1+F'_1+G'_1+H'_1+I'_1)}\right]^2. \tag{5.28}$$

With the exclusion condition of our result it is possible to predict which terms contribute to the intra-Landau-level or inter-Landau-level transitions. This prediction is not clear in other theories. Because of the exclusion condition in the sum of the state in Eqs. (4.20)–(4.27), D_n and I_n terms contribute to the intra-Landau-level transitions from $\alpha=0$ to $\beta=0$, and A_n, B_n, C_n, F_n, G_n, and H_n terms contribute to the neighborhood inter-Landau-level transitions from $\alpha=0$ to $\beta=1$. It is well known that most of transitions arise in those transitions in quantum limit. If $\alpha=0$ and $\beta=2$, all terms contribute to intraband transitions between Landau levels.

In order to compare with other theories [7–11], we separate the energy terms in the denominator of line shape function, and rewrite elements of the line shape function as

$$\begin{aligned}i\gamma_0=&\frac{-i}{\hbar^2\overline{\omega}}\left(\frac{1}{4\pi^2r_0^2}\right)\int_{-\infty}^{\infty}dk_{z\alpha}\int_{-\infty}^{\infty}dq_z\int_0^{\infty}dt\,V(q)^2\Big\{[(N_{\alpha}+1)K_1(N_{\alpha},N_{\beta};t)\Delta F(\alpha)(n_q)]_{(\beta\neq\alpha)}\\&-[[(N_{\beta}+1)(N_{\alpha}+1)]^{1/2}K_2(N_{\alpha},N_{\beta};t)\Delta F(\beta,-q_z)(n_q)]_{(\beta\neq\alpha)}\\&-[[(N_{\beta}+1)(N_{\alpha}+1)]^{1/2}K_2(N_{\alpha},N_{\beta};t)\Delta F(\beta,-q_z)(n_q)]_{(\beta\neq\alpha)}+[(N_{\alpha}+1)K_1(N_{\alpha+1},N_{\beta};t)\Delta F(\alpha)(n_q)]_{(\beta\neq\alpha+1)}\\&+[(N_{\alpha}+1)K_1(N_{\alpha},N_{\beta};t)\Delta F(\alpha)[(n_q)+1]]_{(\beta\neq\alpha)}-[[(N_{\beta}+1)(N_{\alpha}+1)]^{1/2}K_2(N_{\alpha},N_{\beta};t)\Delta F(\beta,+q_z)[(n_q)+1]]_{(\beta\neq\alpha)}\\&-[[(N_{\beta}+1)(N_{\alpha}+1)]^{1/2}K_2(N_{\alpha},N_{\beta};t)\Delta F(\beta,+q_z)[(n_q)+1]]_{(\beta\neq\alpha)}\\&+[(N_{\alpha}+1)K_1(N_{\alpha+1},N_{\beta};t)\Delta F(\alpha)[(n_q)+1]]_{(\beta\neq\alpha+1)}\Big\}\Big/\int_{-\infty}^{\infty}dk_{z\alpha}\,\Delta F(\alpha),\end{aligned} \tag{5.29}$$

$$(i\Delta_1)K_1=\frac{-1}{\hbar^3\overline{\omega}}\frac{1}{4\pi^2r_0^2}\int_{-\infty}^{\infty}dk_{z\alpha}\int_{-\infty}^{\infty}dq_z\int_0^{\infty}dt\, V(q)^2\Bigg\{\Bigg[\frac{(N_\alpha+1)\Delta E_n(\beta,\alpha+1;-q_z)K_1(N_\alpha,N_\beta;t)\Delta F(\alpha)(n_q)}{\hbar\overline{\omega}-\Delta E_n(\beta,\alpha+1;-q_z)+\theta_1}\Bigg]_{(\beta\neq\alpha)}$$
$$-\Bigg[\frac{[(N_\beta+1)(N_\alpha+1)]^{1/2}\Delta E_n(\beta,\alpha+1;-q_z)K_2(N_\alpha,N_\beta;t)\Delta F(\beta,-q_z)(n_q)}{\hbar\overline{\omega}-\Delta E_n(\beta,\alpha+1;-q_z)+\theta_1}\Bigg]_{(\beta\neq\alpha)}$$
$$-\Bigg[\frac{[(N_\beta+1)(N_\alpha+1)]^{1/2}\Delta E_n(\alpha,\beta+1;-q_z)K_2(N_\alpha,N_\beta;t)\Delta F(\beta,-q_z)(n_q)}{\hbar\overline{\omega}-\Delta E_n(\alpha,\beta+1;-q_z)+\theta_2}\Bigg]_{(\beta\neq\alpha)}$$
$$+\Bigg[\frac{(N_\alpha+1)\Delta E_n(\alpha,\beta;-q_z)K_1(N_{\alpha+1},N_\beta;t)\Delta F(\alpha)(n_q)}{\hbar\overline{\omega}-\Delta E_n(\alpha,\beta;-q_z)+\theta_3}\Bigg]_{(\beta\neq\alpha+1)}$$
$$+\Bigg[\frac{(N_\alpha+1)\Delta E_n(\beta,\alpha+1;q_z)K_1(N_\alpha,N_\beta;t)\Delta F(\alpha)[(n_q)+1]}{\hbar\overline{\omega}-\Delta E_n(\beta,\alpha+1;q_z)+\theta_4}\Bigg]_{(\beta\neq\alpha)}$$
$$-\Bigg[\frac{[(N_\beta+1)(N_\alpha+1)]^{1/2}\Delta E_n(\beta,\alpha+1;q_z)K_2(N_\alpha,N_\beta;t)\Delta F(\beta,+q_z)[(n_q)+1]}{\hbar\overline{\omega}-\Delta E_n(\beta,\alpha+1;q_z)+\theta_4}\Bigg]_{(\beta\neq\alpha)}$$
$$-\Bigg[\frac{[(N_\beta+1)(N_\alpha+1)]^{1/2}\Delta E_n(\alpha,\beta+1;q_z)K_2(N_\alpha,N_\beta;t)\Delta F(\beta,+q_z)[(n_q)+1]}{\hbar\overline{\omega}-\Delta E_n(\alpha,\beta+1;q_z)+\theta_5}\Bigg]_{(\beta\neq\alpha)}$$
$$+\Bigg[\frac{(N_\alpha+1)\Delta AE_n(\alpha,\beta;q_z)K_1(N_{\alpha+1},N_\beta;t)\Delta F(\alpha)[(n_q)+1]}{\hbar\overline{\omega}-\Delta E_n(\alpha,\beta;q_z)+\theta_6}\Bigg]_{(\beta\neq\alpha+1)}\Bigg\}\Bigg/\int_{-\infty}^{\infty}dk_{z\alpha}\,\Delta F(\alpha). \tag{5.30}$$

Here

$$\begin{aligned}
\theta_1&\equiv\Delta E_1(\beta,\alpha+1;-q_z)+\hbar\omega_1+\hbar\gamma_1,\\
\theta_2&\equiv\Delta E_1(\alpha,\beta+1;-q_z)+\hbar\omega_1+\hbar\gamma_1,\\
\theta_3&\equiv\Delta E_1(\alpha,\beta;-q_z)+\hbar\omega_1+\hbar\gamma_1,\\
\theta_4&\equiv\Delta E_1(\beta,\alpha+1;q_z)+\hbar\omega_1+\hbar\gamma_1,\\
\theta_5&\equiv\Delta E_1(\alpha,\beta+1;q_z)+\hbar\omega_1+\hbar\gamma_1,\\
\theta_6&\equiv\Delta E_1(\alpha,\beta;q_z)+\hbar\omega_1+\hbar\gamma_1.
\end{aligned}\tag{5.31}$$

We see corrected lineshift terms in Eq. (5.29), which correspond to the first term of the line shape function in Eq. (4.32). These terms are not contained in other theories. The second term of the line shape function in Eq. (5.30) is similar to Ryu and Choi's result [13] and Sawaki's result based on the Stark-ladder representation approach [14]. However, our formula contains more terms expressing the Fermi-Dirac distribution and the complicated energy contribution θ_n.

In this work, θ_n involves complicated interacting energy terms ω_1 and γ_1, which to our knowledge are new terms of this theory. These terms are expected to contribute explanations of experimental data in some moderately strongly interacting systems. If we assume weak interaction and nearest neighbor state transitions, as between $\alpha=0$ and $\beta=1$, we can approximate as follows:

$$\begin{aligned}
\hbar\omega_1&\approx-\Delta E_1(\beta,\alpha+1;\pm q_z)\approx-\Delta E_1(\alpha,\beta+1;\pm q_z)\\
&\approx-\Delta E_1(\alpha,\beta;\pm q_z)
\end{aligned}$$

and

$$\begin{aligned}
\hbar\gamma_1&\approx-\frac{1}{\hbar\overline{\omega}}E_1^2(\beta,\alpha+1;\pm q_z)+\frac{\hbar}{\overline{\omega}}(\omega_1)^2\\
&\approx-\frac{1}{\hbar\overline{\omega}}E_1^2(\alpha,\beta+1;\pm q_z)+\frac{\hbar}{\overline{\omega}}(\omega_1)^2\\
&\approx-\frac{1}{\hbar\overline{\omega}}E_1^2(\alpha,\beta;\pm q_z)+\frac{\hbar}{\overline{\omega}}(\omega_1)^2\approx0.
\end{aligned}$$

Thus we obtain

$$\theta_n\approx0.$$

In order to compare the approximated result of the linewidth with many other theories [7,8,10], we neglect the complicated interaction energy terms θ_n as in the above approximation; at first, we integrate over the vertical component of phonon wave vector, $q_\perp\equiv\sqrt{q_x^2+q_y^2}$, utilizing the relation $\lim_{b\to0_+}(x-ib)^{-1}=p(1/x)+i\pi\delta(x)$. Considering the quantum limit, we obtain the transitions between $\alpha=0$, $\beta=0$, and $\beta=1$. The halfwidth is obtained as

$$\gamma(\omega)\approx R_e\{(i\Delta_1)K_1\}=R_e\{(i\Delta_1')K_1'\}=-\frac{1}{2\pi\hbar^2 v_s}\int_{-\infty}^{\infty}dk_{z\alpha}\int_{-\infty}^{\infty}dq_z\times\Bigg\{V(q_z,q_{\perp 1})^2\sqrt{(q_{\perp 1}^2+q_z^2)}K_1\left(1,0;\frac{r_0^2}{2}q_{\perp 1}^2\right)\Delta F(0)$$
$$\times(n_q)+V(q_z,q_{\perp 2})^2\sqrt{(q_{\perp 2}^2+q_z^2)}K_1\left(0,1;\frac{r_0^2}{2}q_{\perp 2}^2\right)\Delta F(0)(n_q)+V(q_z,q_{\perp 3})^2\sqrt{(q_{\perp 3}^2+q_z^2)}K_2\left(0,1;\frac{r_0^2}{2}q_{\perp 3}^2\right)$$
$$\times\Delta F(1,-q_z)(n_q)+V(q_z,q_{\perp 4})^2\sqrt{(q_{\perp 4}^2+q_z^2)}K_2\left(0,1;\frac{r_0^2}{2}q_{\perp 4}^2\right)\Delta F(1,-q_z)(n_q)$$
$$+V(q_z,q_{\perp 5})^2\sqrt{(q_{\perp 5}^2+q_z^2)}K_1\left(1,0;\frac{r_0^2}{2}q_{\perp 5}^2\right)\Delta F(0)((n_q)+1)$$
$$+V(q_z,q_{\perp 6})^2\sqrt{(q_{\perp 6}^2+q_z^2)}K_1\left(0,1;\frac{r_0^2}{2}q_{\perp 6}^2\right)\Delta F(0)((n_q)+1)+V(q_z,q_{\perp 7})^2\sqrt{(q_{\perp 7}^2+q_z^2)}K_2\left(0,1;\frac{r_0^2}{2}q_{\perp 7}^2\right)$$
$$\times\Delta F(1,+q_z)((n_q)+1)+V(q_z,q_{\perp 8})^2\sqrt{(q_{\perp 8}^2+q_z^2)}K_2\left(0,1;\frac{r_0^2}{2}q_{\perp 8}^2\right)\Delta F(1,+q_z)((n_q)+1)\Bigg\}\Bigg/\int_{-\infty}^{\infty}dk_{z\alpha}(f_1-f_0) \tag{5.32}$$

where

$$q_{\perp 1}\equiv\sqrt{\left\{-\frac{\omega}{v_s}+\frac{\hbar}{m^*v_s}k_{z_\alpha}q_z-\frac{\hbar}{2m^*v_s}q_z^2\right\}^2-q_z^2},$$
$$q_{\perp 2}\equiv q_{\perp 3}\equiv\sqrt{\left\{-\frac{\omega}{v_s}-\frac{\hbar}{m^*v_s}k_{z_\alpha}q_z+\frac{\hbar}{2m^*v_s}q_z^2\right\}^2-q_z^2},$$
$$q_{\perp 4}\equiv\sqrt{\left\{\frac{(-\omega-2\omega_c)}{v_s}+\frac{\hbar}{m^*v_s}k_{z_\alpha}q_z-\frac{\hbar}{2m^*v_s}q_z^2\right\}^2-q_z^2}, \tag{5.33}$$
$$q_{\perp 5}\equiv+\sqrt{\left\{\frac{\omega}{v_s}+\frac{\hbar}{m^*v_s}k_{z_\alpha}q_z+\frac{\hbar}{2m^*v_s}q_z^2\right\}^2-q_z^2},$$
$$q_{\perp 6}\equiv q_{\perp 7}\equiv\sqrt{\left\{\frac{\omega}{v_s}-\frac{\hbar}{m^*v_s}k_{z_\alpha}q_z-\frac{\hbar}{2m^*v_s}q_z^2\right\}^2-q_z^2},$$
$$q_{\perp 8}\equiv\sqrt{\left\{\frac{(\omega+2\omega_c)}{v_s}+\frac{\hbar}{m^*v_s}k_{z_\alpha}q_z+\frac{\hbar}{2m^*v_s}q_z^2\right\}^2-q_z^2}.$$

This approximated result of the linewidth is easier to compare with other theories and experiments through a numerical calculation of double integrations.

VI. CONCLUSION

In this work, at first, we expanded the compact form of CFR scattering factor in the EDPT, reorganized the calculation processes, and presented the calculation processes. We use the EAPT in the EDPT formula. The EAPT has some merits in that it can directly obtain the line shape function and explain the dependence of temperature of line shape function, the dependence of the magnetic field of the line shape function, and other qualities. We strictly used the commutation relation of annihilation and creation operators without using the Kubo identity.

In the calculation, we introduced two useful identities necessary to expand the elements of the scattering factor. We also derived a cancellation relation which simplifies the expansion of the scattering factor. In Eq. (4.32), the main result of this work, we show that the scattering factor (the real part is the halfwidth of the absorption power) contains more interacting effects than some other theories. We expect that this result will explain the experimental data well for some materials. However, in order to examine this result, we need further research for the numerical calculations. From the exclusion condition which results, we show which terms contribute to the intra-Landau-level or inter-Landau-level transitions.

In order to compare with other theories, we separate the energy terms in the denominator of the line shape function. In Eq. (5.29), we separate the correction line shift term which is not contained in the other theories. The second term of the line shape function [Eq. (5.30)], is similar to Ryu and Choi's result [13] and Sawaki's result based on the Stark-ladder representation approach [14]. However, our formula contains more terms expressing the Fermi-Dirac distributuion and a complicated energy contribution in the line shape function. Finally, we summarize the approximation result to compare easily with other approximation theories and experimental results.

ACKNOWLEDGMENTS

This research was supported by the KOSEF (Contract No. 981-0202-009-2), the BSRI program (Contract No. 99-2405) and the Kyungpook University Post-doctoral Program (Contract No. 1999).

APPENDIX

Proof of Eq. (3.11):

$$\mathrm{Tr}^{(e)}\{J^- L_l' \rho_s\} = \mathrm{Tr}^{(e)}\{J^- H_l' \rho_s - J^- \rho_s H_l'\} = \mathrm{Tr}^{(e)}\{\rho_s J^- H_l' - \rho_s H_l' J^-\} = \mathrm{Tr}^{(e)}\{\rho_s [J^-, H_l']\} = -\mathrm{Tr}^{(e)}\{\rho_s [H_l', J^-]\}$$
$$= -\mathrm{Tr}^{(e)}\{\rho_s L_l' J^-\} = -\langle L_l' J^- \rangle_s$$

and

$$\mathrm{Tr}^{(e)}\{J^- L_s L_l' \rho_s\} = \mathrm{Tr}^{(e)}\{J^- L_s H_l' \rho_s - J^- L_s \rho_s H_l'\} = \mathrm{Tr}^{(e)}\{J^- H_s H_l' \rho_s - J^- H_l' \rho_s H_s - J^- H_s \rho_s H_l' + J^- \rho_s H_l H_s'\}$$
$$= \mathrm{Tr}^{(e)}\{\rho_s J^- H_s H_l' - \rho_s H_s J^- H_l' - \rho_s H_l' J^- H_s + \rho_s H_l H_s' J^-\}$$
$$= \mathrm{Tr}^{(e)}\{\rho_s (J^- H_s H_l' - H_s J^- H_l' - H_l' J^- H_s + H_l H_s' J^-)\}$$
$$= \mathrm{Tr}^{(e)}\{\rho_s (-[H_s, J^-] H_l' + H_l' [H_s', J^-])\}$$
$$= \mathrm{Tr}^{(e)}\{\rho_s [H_l', [H_s, J^-]]\} = \langle L_l' L_s J^- \rangle_s$$

Thus we obtain

$$\mathrm{Tr}^{(e)}\{J^- L_1 L_2 \cdots L_{m-1} L_m L_+' \rho_e\} = (-1)^{(m+1)} \langle L_+' L_m L_{m-1} \cdots L_2 L_1 J^- \rangle_e, \tag{A1}$$

[1] R. W. Zwanzig, in *Lectures in Theoretical Physics*, edited by W. E. Downs and J. Downs (Interscience, New York, 1966).

[2] V. M. Kenkre and M. Dresden, Phys. Rev. Lett. **29**, 9 (1971); V. M. Kenkre, Phys. Rev. A **4**, 2327 (1971); **6**, 769 (1972); **7**, 772 (1973); V. M. Kenkre and H. L. Wu, Phys. Lett. A **135**, 120 (1989); P. Grigolini, V. M. Kenkre, and H. L. Wu, Phys. Rev. B **40**, 7045 (1989).

[3] J. R. Barker, J. Phys. C **6**, 2633 (1973); Solid-State Electron. **21**, 261 (1978); J. R. Barker and D. K. Ferry, *ibid.* **23**, 531 (1980).

[4] J. M. Luttinger, Phys. Rev. **121**, 942 (1960); **121**, 1251 (1961); R. S. Fishman, Phys. Rev. B **39**, 2994 (1989); G. D. Mahan, Phys. Rev. **110**, 321 (1984).

[5] W. Xiaoguang, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **34**, 8800 (1986); X. Wu, F. M. Peeters, and J. T. Devreese, *ibid.* **40**, 4090 (1989); X. J. Kong, C. W. Wei, and S. W. Gu, *ibid.* **39**, 3230 (1989); C. S. Ting and X. L. Lei, Solid State Commun. **51**, 553 (1984); X. L. Lei and C. S. Ting, Phys. Rev. B **32**, 1112 (1985); J. L. Birman and C. S. Ting, J. Appl. Phys. **58**, 2270 (1985); C. S. Ting, S. C. Ying, and J. J. Quinn, Phys. Rev. B **16**, 5394 (1977); X. L. Lei and C. S. Ting, *ibid.* **34**, 7003 (1986); D. Y. Xing and C. S. Ting, *ibid.* **35**, 3971 (1987); L. Y. Chen and C. S. Ting, *ibid.* **40**, 3756 (1989).

[6] H. Mori, Prog. Theor. Phys. **34**, 399 (1965); A. Kawabata, J. Phys. Soc. Jpn. **23**, 999 (1967); K. Nagano, T. Karasudani, and H. Okamoto, Prog. Theor. Phys. **63**, 1904 (1980).

[7] S. Fujita and C. C. Chen, Int. J. Theor. Phys. **2**, 59 (1969); S. Fujita and R. Hirota, Phys. Rev. **118**, 6 (1968); A. Lodder and S. Fujita, J. Phys. Soc. Jpn. **25**, 774 (1968); S. Fujita and A. Lodder, Physica B & C **83**, 117 (1976); A. Suzuki and D. Dunn, Phys. Rev. B **25**, 7754 (1982).

[8] P. N. Argyres and J. L. Sigel, Phys. Rev. Lett. **31**, 1397 (1973); Phys. Rev. B **9**, 3197 (1974); **10**, 1139 (1974); S. Badjou and P. N. Argyres, *ibid.* **35**, 5964 (1987).

[9] Joung Young Sug and Sang Don Choi, Phys. Rev. E **55**, 314 (1997).

[10] S. D. Choi and O. H. Chung, Solid State Commun. **46**, 717 (1983); S. N. Yi, J. Y. Ryu, O. H. Chung, J. Y. Sug, and S. D. Choi. Nuovo Cimento D **9**, 927 (1987); Y. J. Lee, S. D. Choi, and S. N. Yi, J. Korean Phys. Soc. **27**, 195 (1994); J. Y. Ryu, S. N. Yi, and S. D. Choi, J. Phys. C **2**, 3515 (1990); Youn Ju Lee, Y. J. Cho, C. H. Choi, Joung Young Sug, and S. D. Choi, Z. Phys. B **98**, 55 (1995); N. L. Kang, Y. J. Cho, and S. D. Choi J. Korean Phys. Soc. **29**, 628 (1996); J. Y. Sug, S. G. Cho, and S. D. Choi, *ibid.* **34**, 135 (1999).

[11] E. Otsuka, T. Ohyama, and K. Murase, J. Phys. Soc. Jpn. **25**, 729 (1968); H. Kobori, T. Ohyama, and E. Otsuka, *ibid.* **59**, 2141 (1989).

[12] Z. X. Cai, S. Sen, and S. D. Mahanti, Phys. Rev. Lett. **16**, 1637 (1992).

[13] J. Y. Ryu and S. D. Choi, Phys. Rev. B **44**, 11 328 (1991).

[14] N. Sawaki, J. Phys. C **16**, 4611 (1983).